Ohmic Circuit Intepretations of Network Distance and Centrality

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Abstract

Ohmic.

Graph-theoretic interpretations of network data have proven popular due in part to their clarity of explanation. The geodesic path length between individuals can be considered to be a measure of closeness between individuals, and the degree of an individual is a proxy for both gregariousness and popularity. Additionally, statistical measures for characteristics of the whole ensemble, such as global transitivity and reciprocity, may model the tendencies of the network's individuals to form particular relationships, given other relationships present in the system.

However, there are several shortcomings for this method of analysis that cannot easily be rectified. Geodesic path lengths may not give an accurate measure of social distance if multiple shortest paths exist, such as the case where two unconnected people have one mutual friend, or ten. In order to measure the importance of a tie rather than a node, a measure of betweenness can be constructed using geodesic paths [Freeman, 1979], though the issue of multiple geodesic paths, possibly sharing edges between them, requires solutions that may prove difficult to interpret.

As an alternative, there is another well-studied model where multiple paths between nodes have a straightforward interpretation. When considering social ties as conduits for information transfer, there is an immediate analogue to electrical circuitry: the connection points for circuit components are seen to be the nodes of a network, and a signal is sent from one node to another in the form of an applied voltage difference. The strength of this signal, in terms of the current flow that results, depends both on the properties of each edge in the system as well as their topological arrangement.

The rest of this chapter proceeds as follows. First, there is a brief outline of the concept of electrical conductance, in terms of a fixed potential difference applied across a pair of nodes. It is then demonstrated how this measure allows us to determine a refined measure of social distance between two individuals in a network, as well as the relative importance of a tie for information transfer using an analogue of electrical power. Following this, a possible solution for including antagonistic relationships is proposed using a slight modification of the Ohmic framework.

1 Previous Work

Approaches for considering electrical circuits as networks are at the root of Kirchhoff's Circuit Laws, discussed in detail in Section 1.2, so it is not surprising that the extension to social networks has been considered repeatedly in the past 150 years. Among these considerations are those by Freeman [1979], in which degree-based and betweenness centrality are discussed. These were natural precursors to Freeman et al. [1991], which dealt with the notion of network "flow" as a method of connectivity between two points, in which the value of a network tie is taken to be a flow capacity – essentially, that all ties in a network have equal length but varying diameter.

Circuit-law based derivations are found by Stephenson and Zelen [1989], and rediscovered in the form of random walks on a graph by Newman [2005]. The underlying methods for calculating current flow are essentially identical to those presented in this paper, as they are based on the Kirchhoff methodology, but with differing interpretations of betweenness and little mention of distance; these concepts are elaborated on in this work.

1.1 Social Pathways, Milgram's Experiment and the Flow of Information

As an oft-mentioned study of cross-country sociology and community, Stanley Milgram's small-world experiment [Milgram, 1967] was aimed to determine the minimum number of connections between individuals separated by geography. The experiment had a number of people from Nebraska try to send a package to a person in Boston, with the constraint that the package could only be passed to an intermediary known personally by each respective sender in the chain. [1]

One crucial piece of this experiment is that each person in the experiment, with the exception of the final recipient, was able to communicate only through one more person. As a result, only single paths were traced between the originators and the recipient. While this model may be useful for demonstrating a short distance in social space along a single path, it does not accurately represent the mechanism highlighted by the experiment – the transfer of information along social ties. Replication, division and transmission of information are qualities that cannot be duplicated in an experiment with this physical limitation.

As a thought experiment, consider what could happen if the small world experiment were repeated, but where each source of information were given, say, one million small packages to send to the target in Boston. The previous goal, that they attempt to have the transmission take as few paths as possible, remains in effect, though there are also practical problems with sending all the packages to one local recipient for fear of a backlog problem. Over time, each recipient can report back to the original sender with their ultimate capacity and how much traffic they can handle; this

[1] The observed average of the number of senders was roughly six, yielding the popular aphorism "Six Degrees of Separation", a point made in detail throughout the networks literature.

is then taken into account by the original sender as they plan for future deliveries.

This is a crude but accurate representation of the flow of current in an electrical circuit. Resistance to current represents the energy required for a signal to be transmitted at every step, and nodes have a potential energy level that governs the degree to which current flows. As seen in Figure 1, this model illustrates the importance of each node and edge in the flow of information, with a natural extension to edges that have greater transmission power than others.

1.2 Ohmic Circuits and Kirchhoff's Laws

This method of exploring networks requires three quick definitions from the circuit theory literature. Electric current I is the result of an electric field being applied to a conductor, in which particles of one charge are free to move and others are fixed in place, and is defined as the rate at which charge moves past a reference point per unit of time.

> Kirchhoff's Current Law states that charge cannot accumulate at any point in a completed circuit. This implies that with respect to any fixed point, the net current is zero; that is, any current going in must also be balanced by an equal amount of current going out.

The energy carried by an electric current is most often expressed in terms of an "electric potential difference" or "voltage" V, which is equal to the amount of energy transferred per unit of charge.

> Kirchhoff's Voltage Law states that in any closed loop, the total electric potential difference must be zero. As a corollary, any two paths taken from one point in a circuit to another must have identical potential differences.

Ohm's Law relates the behaviour of current to that of potential difference along a circuit element:

> The ratio of current to potential difference, I/V, is called the conductance, with symbol G. In so-called Ohmic circuits, a conductor is a device for which this ratio of current to potential difference is maintained for all feasible potential differences across its length. [2]

From these three pieces of information, one can define the equivalent conductance of an arrangement of conductors in terms of the current produced by a specified applied potential difference (as in, say, a common household battery). By determining the total current I_{total}, it is possible to calculate the equivalent conductance of the circuit as $G_{eq} = I_{total}/V_{total}$.

[2] This is usually represented canonically as the "resistance", or R, defined as the reciprocal of conductance. I use conductance in this paper for its isomorphism to the strength of a connection, which in the sense of information takes the form of current.

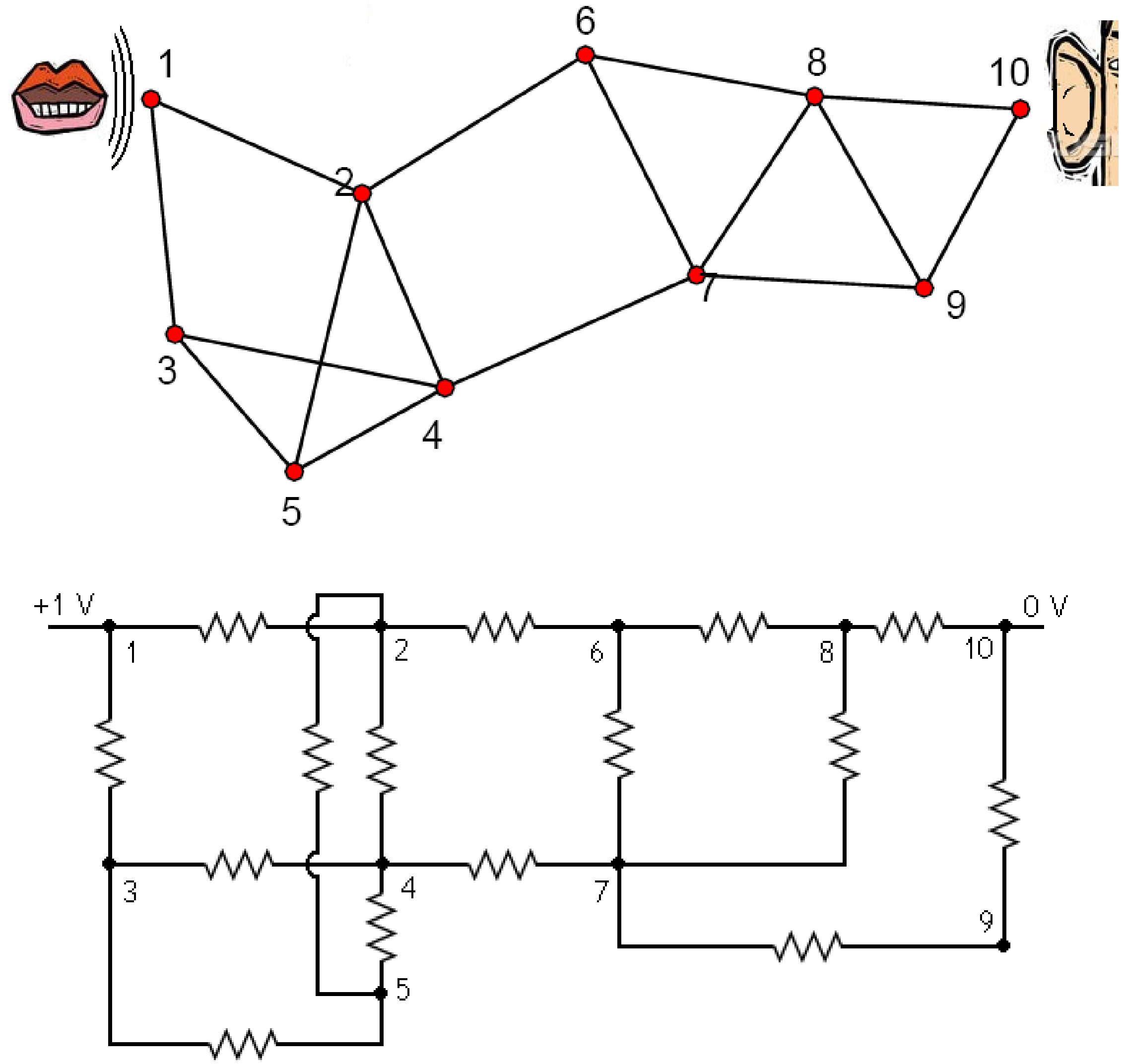


Figure 1: Representing the analogy between information transmission through a social network and current flow through a compound electrical resistor.

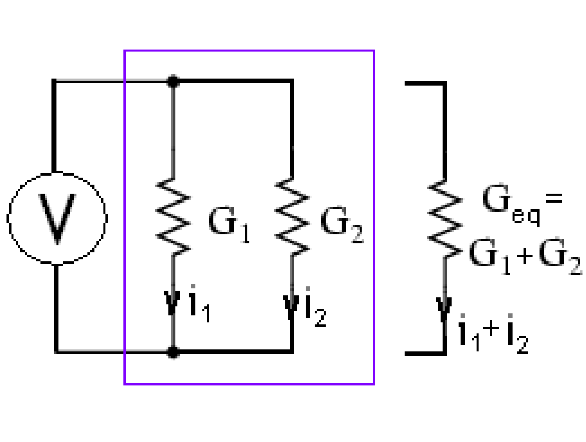


Figure 2: Simple circuits demonstrate compound conductances. In the parallel case on the left, the total conductance is the sum of the respective conductances because the induced current is higher; in the serial case on the right, the total conductance is the reciprocal sum due to the drop in induced current.

In the case of two parallel conductors attached to a common potential difference, the total current is found to be

$$I_{total} = VG_1 + VG_2 = V(G_1 + G_2),$$

yielding the equivalent conductance of two parallel conductors as their sum, $G_{eq} = G_1 + G_2$.

In the case of two serial conductors, note that the total potential difference across the conductors must equal that of the source:

$$V = \frac{I}{G_1} + \frac{I}{G_2} = I\left(\frac{1}{G_1} + \frac{1}{G_2}\right);$$

this yields the equivalent conductance, given by $1/G_{eq} = V/I = (1/G_1 + 1/G_2)$.

Similar expansions can be calculated for more complicated circuit arrangements, but the equivalent conductance for any pair of nodes, even those without direct connections, can be calculated using the following algorithm:

1. Create a vector $(V_1, ...V_n)$ specifying each intersection node in the system.

2. Choose the nodes (a, b) across which the equivalent conductance is to be found. Set $V_a = 1$ and $V_b = 0$.

3. Set up a system of equations using Kirchhoff's Current Law and Ohm's Law for all nodes (except the source and sink nodes a and b):

$$0 = I_k = \sum_{j \neq k} \frac{(V_j - V_k)}{G_{jk}}$$

Note that the currents through the source and sink nodes, I_a and I_b, are by definition non-zero, reflecting the induced current due to the applied potential difference across the circuit.

4. Solve this system of equations for the remaining V_k, noting that all values must be in the interval $[0, 1]$.

5. Determine the total induced current

$$I_a = \sum_{j \neq a} (1 - V_j) G_{aj}$$

and set $G_{eq} = \frac{I_a}{V_a - V_b} = I_a$.

As this yields a set of electric potentials, Ohm's Law identifies the current across any conductor as $I_{jk} = (V_j - V_k)G_{jk}$.

1.2.1 A Note of Comparison

The method suggested here is identical in solution to that proposed by Stephenson and Zelen [1989] and Newman [2005], except that these works suggest a fixed current approach rather than a fixed voltage. In fact, as the voltage-based method requires a matrix inversion for every node pair considered, it is less than practical for a full analysis of a system with more than 200 nodes on currently available computing hardware, and the implementations in this work have been computed using fixed current.

For the sake of the analyses that follow, both the fixed-voltage and fixed-current approaches will be used to investigate networks. Since the relationship between voltage and current is linear, the results only differ by a factor of the equivalent conductance and are immediately deriveable from each other.

2 Social Conductance

In a social network setting, when individuals are considered to be nodes in a graph, the absence or presence of an edge determines the ability to conduct information directly between the individuals in question. Previous explorations have considered "degree", or the minimum path length between two individuals, to represent the effective distance between individuals; this representation assumes, however, that information can not be replicated and sent separately along multiple channels.

As a result, define **social conductance** as being the rate at which two individuals can share information directly between them, and **equivalent social conductance** as the total information flow rate between individuals when accounting for all possible paths of transmission and conductance. In the case of a binary network, the social conductance of a tie is set as equal to one, and for the lack of a tie is equal to zero. It must also be noted that there is an immediate extension to univariate non-binary relations, since any nonnegative tie strength can be represented as a social conductance.

Once the potentials at each node in the system are determined, the current flow across any edge can be measured given the applied potential difference across the two nodes. This current then represents a fraction of information that travels from one person to another within the network, and are internally comparable for determining which edges are the most influential in conducting information. See Figure 4 for a comparison of several paths.

2.1 A Measure of the Effective "In-Network" Strength of a Tie

For each pair of nodes in the network, the equivalent social conductance is determined using the aforementioned procedure. As a measure of social connectivity, this now represents not only the strength of a direct connection but of all social pathways connecting two individuals. This interpretation immediately lends credence to Simmel's hypothesis [Simmel, 1955] that influence through a single tie is insufficient to capture sociological phenomena without considering common connections. Because this measure considers the entire network when calculating the degree of connection between two individuals, this effectively extends the measurement of social distance to the consideration of multiple middlemen alongside a relationship.

To demonstrate, consider the effective tie strength between two members of an n-clique. As demonstrated in Figure 3, when inducing a potential difference across any edge, the remaining points each take an electric potential halfway between the source and sink nodes. Because no current flows between points of equal potential, the only remaining edges that carry current are the direct path and the remaining $n-2$ two-step paths, all in parallel. As the conductance of two serial conductors is half that of each component, the total conductance of the remaining assembly is $G + \frac{n-2}{2}G = \frac{nG}{2}$.

It follows immediately that the observation of a binary tie within a highly interconnected component may in fact be illusory; that this is in fact a long-distance tie in true magnitude that is only observed due to the influence of their intermediaries. This "local amplification" effect is purely endogenous, making it difficult to disentangle in the case of binary tie observation.

An additional visual comparison between the geodesic and Ohmic-social models is shown in

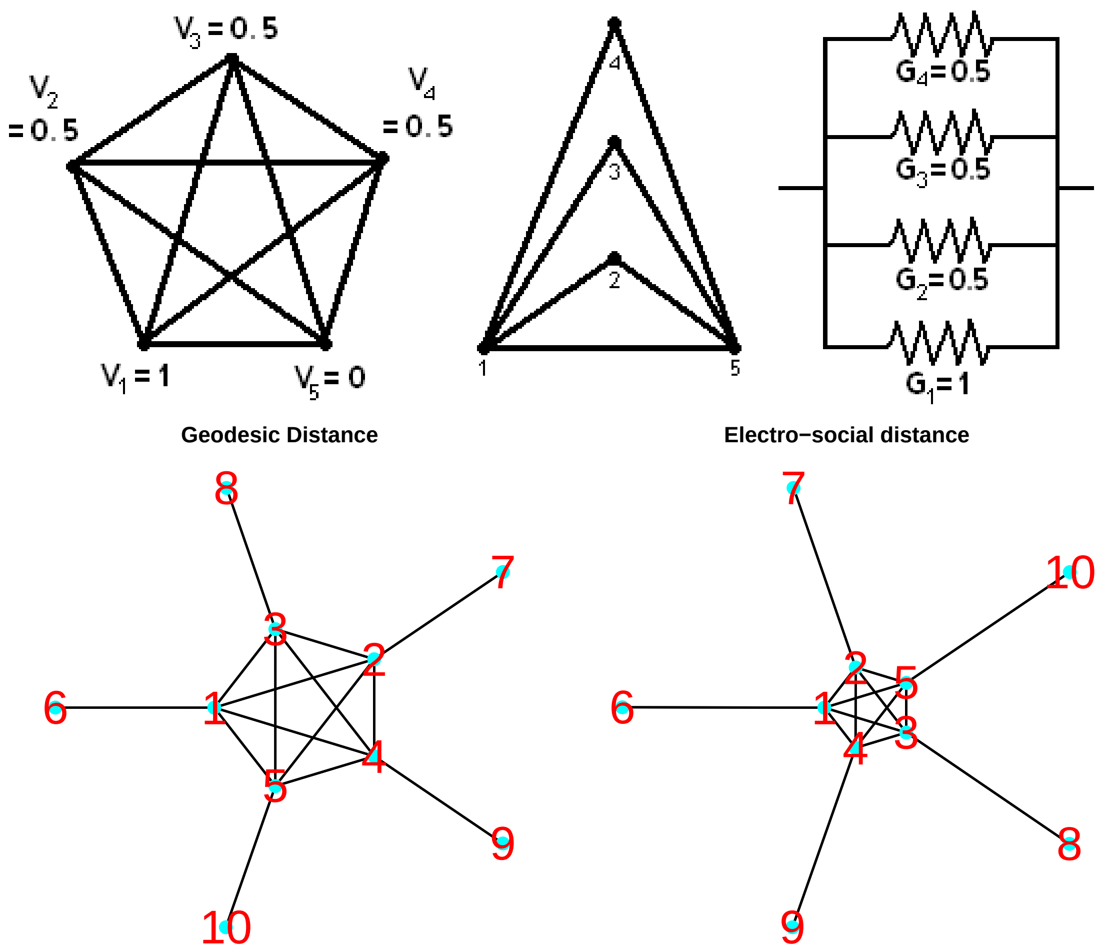


Figure 3: Above, the equivalent social conductance between members of a five-clique. As additional middlemen are added, the conductance increases, reinforcing the theory of Simmelian ties. Below, a visual comparison of social distance under the geodesic and Ohmic-social models.

Figure 3, in which five individuals form a clique but each maintain one outside friend. If the underlying tie strengths are all identical, the geodesic method does not consider the outside friendships to be any closer than those within the clique, whereas the social conductance method allows for this possibility.

2.2 Tie and Node Importance As Average Observed Current

Consider that with a fixed potential difference across nodes a and b, there is an induced potential at each node in the graph V_i between 0 and 1, and an induced current between the nodes specified as

$$I_{ij}^{ab} = (V_i^{ab} - V_j^{ab})G_{ij}.$$

If the total induced current in the system is I^{ab}, define the **embedded importance** of an edge in this configuration as the fraction

$$M_{ij}^{ab} = \frac{I_{ij}^{ab}}{I^{ab}},$$

and the average embedded importance of an edge as

$$M_{ij} = \frac{1}{\binom{n}{2}} \sum_{a} \sum_{b \neq a} M_{ij}^{ab}.$$

This is defined with respect to directed edges, so that one arc in a dyad may be more important than another for whatever reason of topology, as discussed ahead.

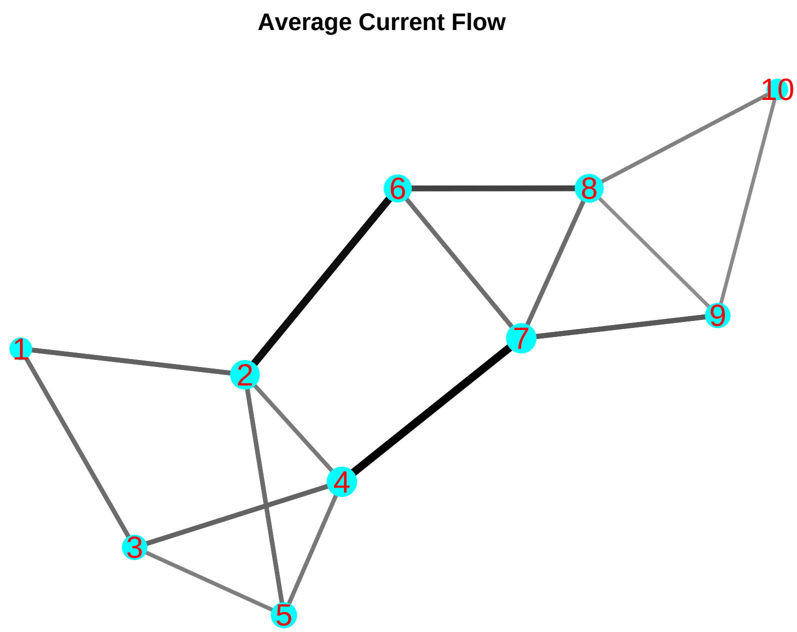
This measure is *not necessarily* equal to the drop in current that would be observed if the tie were removed from the system, defined as the **tie-cut importance**,

$$T_{ij}^{ab} = \frac{1 - (I_{ab}|G_{ij} = 0)}{I_{ab}|G_{ij} = G_{ij}(true)};$$

in fact, as part of this current may be directed to other less-used nodes, it can be shown that the embedded importance is an upper bound on the true tie-cut importance, $M_{ij}^{ab} \geq T_{ij}^{ab}$.

As a comparative measure, the embedded importance has many desirable properties. It can be calculated simultaneously for all ties in the system given a source-sink pair, whereas tie-cut importance requires at most $2\binom{n}{2}$ calculations, one for each tie to be removed. Additionally, the relative order of tie importance will be extremely close in most circumstances.

To demonstrate, Figure 4 shows the sample network from Figure 1 with potential differences applied across pairs of nodes, where line and node thicknesses represent current flows and therefore importances to information transfer.



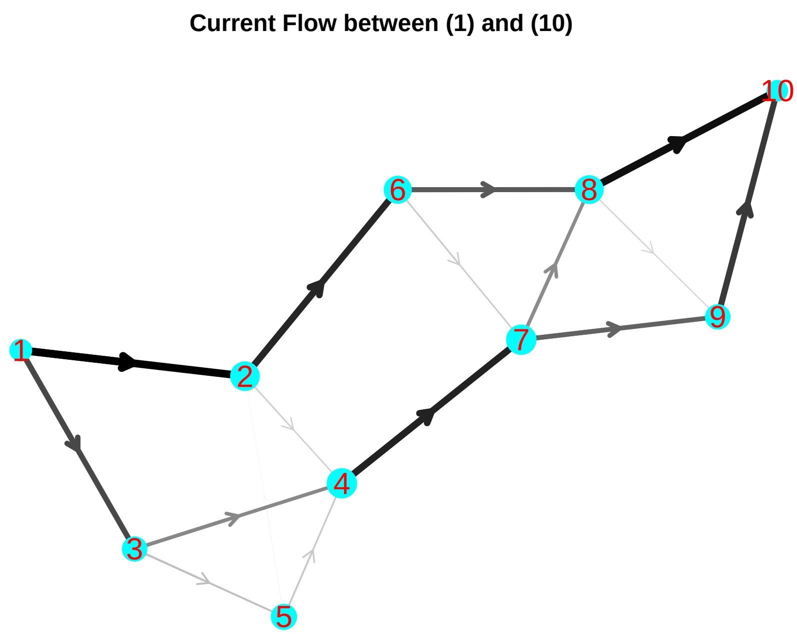


Figure 4: Current through nodes and edges of a sample network. Line thickness represents the fraction of current going through an edge; node size represents the total current flowing into (and out of) the node. Top: The current along each edge of the network, averaging together every pair of source and sink. Bottom: The current flow for the source at node 1 and sink at node 10.

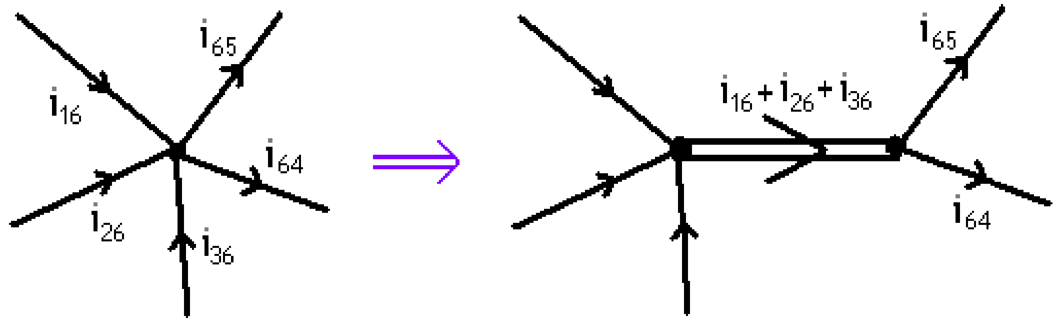


Figure 5: Current through a node as importance of the node in communication.

3 Betweenness Measures For Nodes Based On Current Flow

The concepts in the previous section to evaluate edge importance can be extended immediately to nodes by taking the sum of all currents entering or leaving a node during an Ohmic-social path test, as seen in Figure 5.

The measure of betweenness centrality for a node, as defined in Freeman [1979], is an average of the fraction of shortest paths between two other nodes that contain the third node. Since non-geodesic paths may also play an important role in node-to-node communication, an approach that considers multiple paths appropriately weighted can be valuable. Current-based measures for centrality are the prime focus of Newman [2005], and occupy a full chapter of Bollobas [1998].

The nature of how the averaging is conducted, however, is paramount to the calculation of a centrality statistic. The assumptions of Newman [2005]; Bollobas [1998] are that each node pair has equal weight when composing betweenness centrality; that is, that a pair of nodes of large geodesic distance (say, 12 intermediate nodes) carry as much weight for calculating betweenness as two adjacent points.

By considering equivalent distances based on a voltage measure, the current produced effectively decreases with distance, though it also multiplies with the number of paths between targets. Therefore, care must be taken when applying a measure of betweenness based on current flow. Three Ohmic betweenness measures are presented, based on what quantity should be constant over averaging for each node-pair trial: current flow (I=1), potential difference(V=1), or electrical power (VI=1).

Having defined the current through edge (i,j) as

$$I_{ij}^{ab} = (V_i^{ab} - V_j^{ab})\mathbb{I}(V_i^{ab} > V_j^{ab})G_{ij}$$

with respect to terminal potentials ($V_a = 1, V_b = 0$) and a total induced current I_a, the centrality measures are defined with respect to these terms.

The notation of Freeman [1979] gives $C_D(i)$, $C_C(i)$ and $C_B(i)$ as the degree, closeness and (geodesic/shortest path) betweenness centralities respectively for a node labelled i.

3.0.1 Fixed-Voltage Ohmic Betweenness Centrality

Define $C_V(i)$ as the centrality of node i as determined by the sum of the current flowing from it, averaged with respect to an applied voltage of 1 across all pairs of nodes:

$$C_V(i) = \sum_a \sum_{b \neq a} \sum_{j \neq i} I_{ij}^{ab}.$$

3.0.2 Fixed-Current Ohmic Betweenness Centrality

Define $C_I(i)$ as the centrality of node i as determined by the sum of the current flowing from it, averaged with respect to an applied current of 1 across all pairs of nodes:

$$C_I(i) = \sum_a \sum_{b \neq a} \frac{1}{G_{ab}^{eq}} \sum_{j \neq i} I_{ij}^{ab}.$$

3.0.3 Fixed-Power Ohmic Betweenness Centrality

The power through an electrical circuit is equal to the current passing through it multiplied by the potential difference, or $P = VI$; factoring in Ohm's Law, $P = V^2 G$. A unit power is achieved by setting the potential difference to $V_a = \frac{1}{\sqrt{G_e q}}$. This yields

$$C_P(i) = \sum_a \sum_{b \neq a} \frac{1}{\sqrt{G_{ab}^{eq}}} \sum_{j \neq i} I_{ij}^{ab}.$$

The choice of class of Ohmic betweenness centrality depends on the application at hand, though each has a natural explanation: in order, the multiplicative effect of parallel paths, the effect of larger distances requiring more connections, and a constant amount of energy for any individual connection.

4 Extension to Directed Graphs

The above methods have derived from standard direct current circuit theory, in which the conductance in each direction is equivalent. To apply this method to directed graphs, including those where conductance in each direction is non-zero but not necessarily equal, consider the notion of a

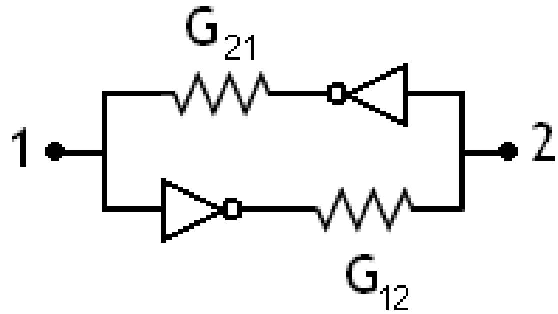


Figure 6: A differential resistor, with diodes set up to enforce a one-way flow of current; if current flows from node 1 to 2, the conductance equals G_{12} and current only goes through the bottom path; likewise, the conductance is G_{21} if current flows in that direction.

differential resistor, in which the resistance/conductance of an edge in the system depends on the direction of current; see Figure 6 for a sample electrical diagram.

Because this introduces a non-linearily into the system, a single algebra operation will be unable to solve for the state of the system. However, because current is continuous with respect to voltage in this element (though non-differentiable at zero), the following iterative procedure can solve for the equilibrium voltages and currents:

1. For each asymmetric edge, create a vector of indicators whether current should flow from the lower-numbered node to the higher one, or vice versa. (As a default, set all to be "ascending" – however, this method can be sped up by cleverly guessing which elements will have current flow in each direction in advance.)

2. Create a symmetric sociomatrix by replacing all asymmetric elements with those for the indicated flow direction, and solve this system to obtain the equilibrium voltages.

3. Determine whether any of the true current flows violate their assigned path by examining the differences in voltages in the asymmetric edges.

4. If there are no conflicts, the procedure is finished; if there are, reverse the incorrect flows in the indicator vector, and repeat the previous two steps (and this one) until equilibrium is reached.

All previous measures considered for nodes and edges can still apply, since with each potential difference applied, the system behaves as if it were a traditional Ohmic circuit.

5 Extension to Stochastic Relational Data

The purpose of assembling this toolkit has been to evaluate practical tie-level statistics under conditions when a tie strength is stochastic in nature. Given an (exogenous) generative model for tie strengths, a set of networks can be drawn from the underlying parameters. This produces a joint distribution of ties that can be examined for their relevant statistical observations.

For example, the most "important" tie in one instance of a network may prove to be that way due entirely to the underlying uncertainty, and that other ties may prove to be just as important in other instances. Rank-based measures, such as centrality and edge importance, can immediately be compared across separate instances of a generative graph process.

6 Reverse Engineering: Solving for Marginal Tie Strengths

It has previously been noted that these methods are calculated with respect to true underlying tie strengths, yet in many situations the data that are collected do not represent a true dyadic interaction.

Inherently, measures of association based on higher-order observations, such as children in the same class, reflect a multitude of contained dyadic relationships, as well as higher-order polyadic relationships (triads, tetrads, etc.) that exist. Rather than observing a dyadic data point, each relationship measurement is made conditional on every other relationship.

In order to disentangle the underlying marginal ties, independent of other levels of communication, it is possible to estimate particular configurations using the following method:

1. Assume that all observations are the result of dyadic interactions only. (In general, assume that all dyadic observations result from polyadic social structures of unspecified dependence.)

2. Specify an underlying dyadic configuration, and calculate the effective distances between nodes.

3. Calculate the differences between the model and observed distances, and determine the energy for the total distances under a chosen energy configuration, such as the Kamada-Kawai spring-embedding method [Kamada and Kawai, 1989].

4. Modify the underlying structure so as to minimize the energy difference between the configurations; repeat until a maximum has been reached.

This is similar to calculating the partial correlation for a collection of associated variables to determine the underlying structure. An ensemble of graphs can then be assembled to approximate a generative distribution, with weights proportional to the respective energy differences.

7 Anti-Social Conductance? Estimating Social Distance In The Presence of Antagonistic Relationships

While the Ohmic conductance approach discussed to this point is able to consider ties of any positive value, the method cannot be immediately extended to antagonistic relationships while perfectly maintaining the metaphor. By considering the flow of current as a means of propagating a message from a source to a target, one can model a "negative tie" as a connection where the message is transformed rather than reversed in direction. It is then shown how this method allows for effective distances between two individuals to vary above or below the underlying dyadic distance.

Consider the example in Figure 7, which shows a hypothetical social network of 10 people where (Ben, Frances) and (Icarus, Jessica) have antagonistic relationships. Rather than act as positive influences, these ties have the effect of destabilizing the network, weakening otherwise friendly relations and pushing them apart; this is apparent in the tie between (Donna, Germaine), who are parallel to (Ben, Frances), and straining Holly's friendships to both Icarus and Jessica.

7.1 Enemy Metaphors: Messages In Bottles, or Social Chromodynamics

The concept of social conductance uses two metaphors to describe social interaction: circuit pathways, which replace a social tie with an electrical conductance; and signal propagation, such that the flow of electric current represents the transmission of information, and that larger conductances represent stronger signals being transmitted.

Consider the notion that current represents the flow of information. If this is in the form of a message, the only variation would be in the strength of the signal rather than the content. If electric current is substituted with another type of flow, one with similar physical properties but otherwise transmutable, the current analogy can be augmented with a quantity describing signal content.

One interpretation is that current flow is represented by the transmission of a binary signal within a physical medium. Among the allegories that fit this model are:

- The outcome of a fair coin flip, heads or tails.
- A declaration of feeling: "She loves me, she loves me not."

A friendly tie is described as one that transmits each signal it receives with perfect clarity; in an "enemy" tie, the signal transmitted is the opposite of the one received; see Figure 8 for a demonstration. In aggregate, a node will receive a collection of signals from nodes with higher electric potential, and transmit to nodes with lower electric potential in a degree proportional

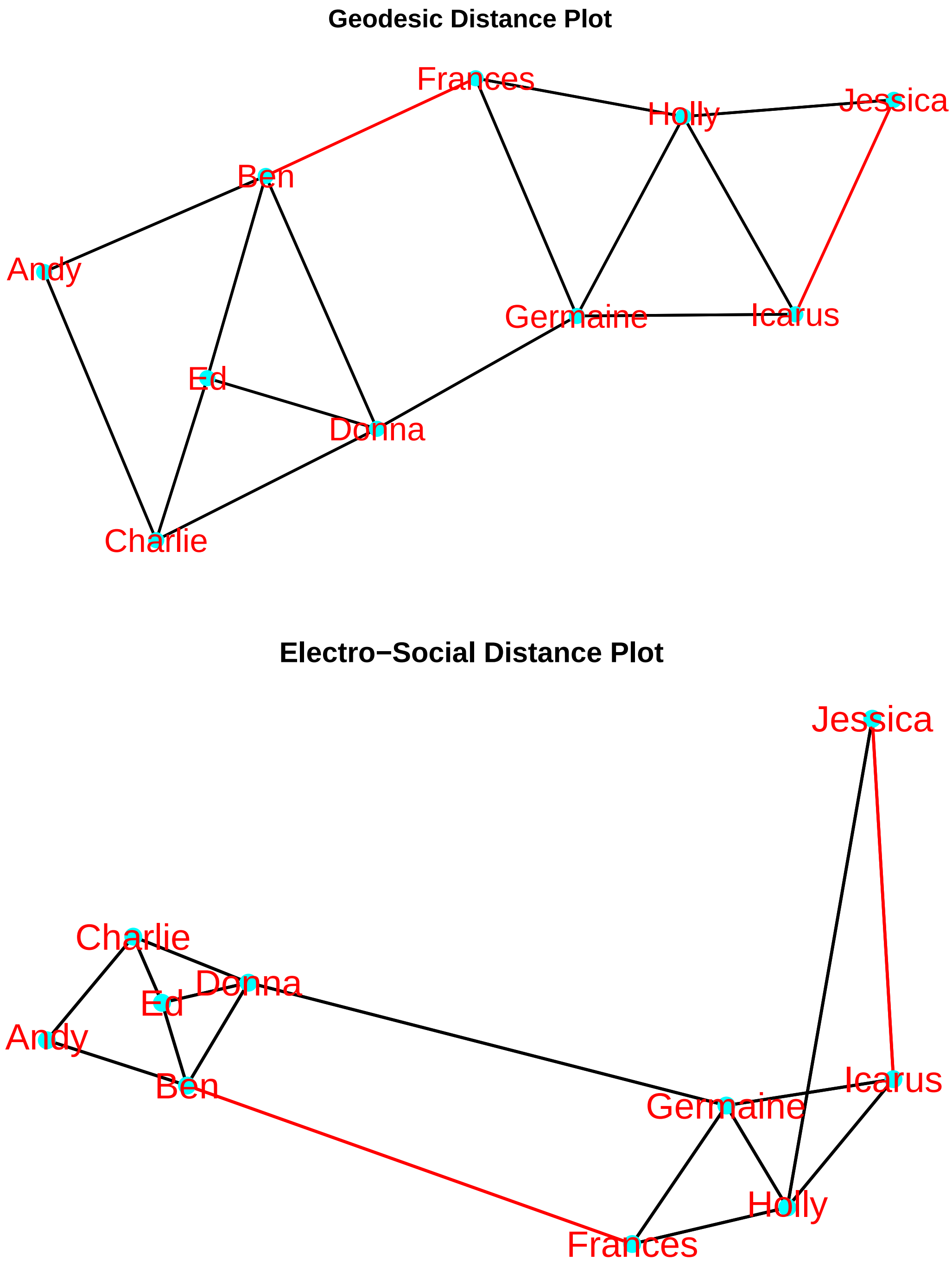


Figure 7: Two plots of a hypothetical ten-node social network with two antagonistic connections. Above, the distances between nodes represent geodesic path lengths; below, distances are calculated as the inverse of effective social conductance, divided by the absolute value of the signal fidelity.

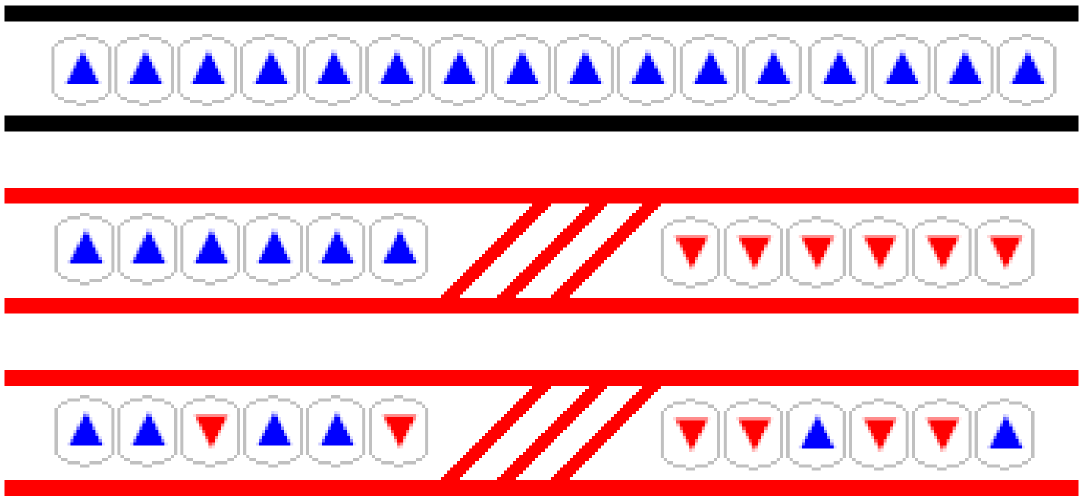


Figure 8: Signal transmission through "friend" and "enemy" connections. In the friend case (uppermost), signals are transmitted with perfect fidelity. Enemy connections reverse all signals, as shown in the bottom two connections.

to both the potential difference and the conductance between the source node and its respective targets; the signal transmitted will either be as received, to friendly connections, or flipped around to enemies. [3]

For a relational data system consisting of "friend" and "enemy" ties of varying magnitudes, the total current flow will be identical to one where the system contains identical ties magnitudes, but only friendly ties. The only modification will be in the nature of the signal received in the end.

7.2 Definitions: Social Conductance in Partly Antagonistic Networks

The method described so far comes about by applying a fixed potential difference across two nodes and calculating the resulting current flow; for a unit value of potential difference, the equivalent conductance is equal in magnitude to the current produced. In this case, there is an additional element: what fraction of the current received by the target is opposite that which was sent initially.

Given nodes i and j, let (B_{ij}, R_{ij}) be the current received at node j of each type[4]. Define:

- Strength of Signal $I_{ij} = B_{ij} + R_{ij}$, or the total current flowing from node i
- Fidelity of Signal, $f_{ij} = \frac{B_{ij}-R_{ij}}{B_{ij}+R_{ij}}$. Equal current of each type results in a message consisting entirely of noise, since the signals offset each other completely.

[3]Clearly, this analogy is not perfectly suited to true social communication; over time, messages from enemies would be ignored once the relationship became clear to both parties. However, its properties are compelling enough that I am willing to proceed with it for the purposes of prediction.

[4]For lack of better symbology, let B and R represent "blue" and "red" respectively for each type of signal, as coded in the diagrams.

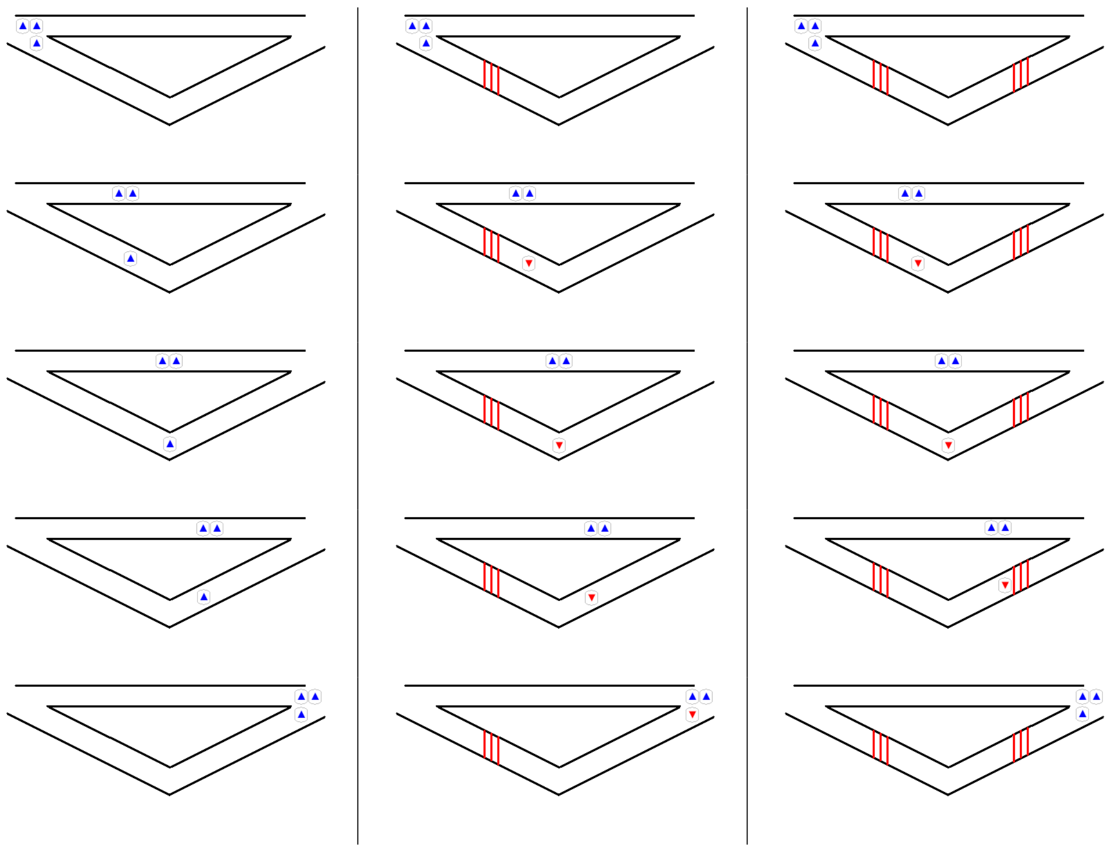


Figure 9: Social conductance with friend-of-friend, friend-of-enemy and enemy-of-enemy triads. Friend-of-friend connections maintain an increased signal strength and perfect fidelity, while one enemy connection on the indirect path causes the signal fidelity to decrease. With an enemy-of-enemy connection, the signal flip is undone, restoring an identical signal fidelty as in the friend of friend case.

From this definition, social distance measures can be constructed; one possibility is that the equivalent social conductance $\bar{C}_{ij}$ is defined as $I_{ij} f_{ij}$, and social distance as defined as the inverse of that measure.

7.3 Examples of Triads

As is shown in Figure friend-friend, there are three forms of triad that deserve immediate description: how an existing friendship is affected by a third party, and this party's respective relationships in forming a triad.

In the case where the third person is on good terms with both partners, the effective strength of the tie increases. If the underlying friendships all have social conductance G, it is as if each relationship is composed of a direct tie G plus an indirect tie of strength $G/2$, for a total strength $3G/2$.

Supposing that the third party has one enemy and one friend in the triad, the signal strength is maintained at $3G/2$, but the effective fidelity of the connection is $1/2$, reducing the effective social conductance to $3G/4$ and lowering the strength of the original friendship. It is also worth noting that the direct enemy connection is itself mitigated, decreasing in magnitude from an original conductance of $-G$ to a lesser $-3G/4$.

In the case where the original two parties share a common enemy, any negations along pathways are cancelled, leaving an equivalent social conductance of $3G/2$ as before.

7.4 Algorithm for a Complete Social Network

Since the total current flow between two nodes is identical in the cases where ties are friendly or antagonistic in nature, the effective strength and fidelity of a tie can be determined through a two-step process. First, the effective tie strength is calculated as if all ties in the system are friendly. Then, fidelity is calculated using the following method:

- Determine the electric potential levels at each node, as calculated from the tie strength algorithm, and note their order from highest (the source) to lowest (the sink).
- Calculate the total current flowing out of the first node, in terms of both upright and inverted currents.
- For each outgoing current from the node, replace the outgoing current with a sum of upright and inverted current equal in magnitude to the original signal and in proportion to the balance in the total current.
- For each outgoing current where the tie is antagonistic, switch the proportions of upright and inverted currents.

- Repeat these steps for each node in order of descending electric potential.

Only one pass is necessary to solve for the current balance in each edge, and therefore for each source-sink pair in the system.

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